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Brief Paper

A Novel Iterative Learning Control Formulation of Generalized Predictive Control*

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Key Words—Predictive control; learning control; iterative methods; robot control; manufacturing processes.

Abstract—A novel iterative learning control formulation of the long-range predictive control method generalized predictive control (GPC) termed GPC with learning (GPCL) is developed. GPCL is applicable to processes that are repetitive in nature and are subject to a partially repeatable disturbance. It improves the regulation performance of GPC by learning and compensating in advance for the repeatable portion of the disturbance over a series of trials. GPCL is proven to have the same stability properties as GPC, and to converge asymptotically on condition that its feedback loop is stable. According to a simulation study, for six plants of varying control difficulty, GPCL outperformed GPC after two trials, and reduced the final value of the output variance by 48% on average for a 50% repeatable disturbance. In robotic edge-following experiments with a 47% repeatable disturbance, GPCL reduced the average output variance by 32% relative to GPC after three trials. After ten trials, the reduction was 43%.

1. *Introduction*

In recent years, a number of adaptive long-range predictive control (LRPC) methods have been proposed. These algorithms have been shown to be less sensitive to plants with variable or unknown dead-time or with unknown order than earlier self-tuning approaches, and are capable of controlling both non-minimum-phase and open-loop-unstable plants (De Keyser *et al.*, 1988). This paper describes a novel formulation of the LRPC algorithm generalized predictive control (GPC) (Clarke *et al.*, 1987a) for iterative learning applications. The results obtained here for GPC are also valid for the algorithms that are obtained as subsets of GPC by choosing particular output and control horizons (such as GMV and EPSAC).

In iterative learning control schemes the plant inputs are modified over a sequence of repetitions or trials with the objective that the plant outputs converge to a predetermined reference trajectory. Extensive surveys of research in this field are given by Moore *et al.* (1992) and Horowitz (1993). These schemes are applicable to processes that are repetitive in nature and operate over a fixed time interval, such as mechanical systems in a manufacturing environment or batch process control.

In this paper a novel iterative learning control formulation of GPC is developed. This formulation is termed generalized predictive control with learning (GPCL). Rather than altering the plant inputs to converge on a predetermined reference trajectory, the repeatable portion of the disturbance is learnt over the series of trials and used to improve the accuracy of the output predictions. The controller is then able to compensate at the current time for the predicted effect of future disturbances, yielding tighter output regulation. The improved predictions have the additional benefit of often reducing the amount of control action required. Unlike many learning algorithms, the GPCL approach may be used in combination with on-line adaptive control, and does not require the reference trajectory to be constant over the trials. It utilizes a linear plant model, and is therefore only applicable to processes that can be locally linearized about an operating point. The design of the control law is described first, followed by an analysis of its convergence, a comparative simulation study and an application example.

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2. *The control law*

2.1. *The plant model and prediction equations.* GPC and GPCL are both based on a controlled auto-regressive and integrated moving-average (CARIMA) plant model:

$$A(q^{-1})\Delta y(t) = q^{-1}B(q^{-1})\Delta u(t) + C(q^{-1})\xi(t), \qquad (1)$$

where

$$A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_{na}q^{-na},$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \dots + b_{nb}q^{-nb},$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \dots + c_{nc}q^{-nc},$$

$y(t)$ is the plant output, $u(t)$ is the plant input, $\{\xi(t)\}$ is a zero-mean disturbance, q^{-1} is the backward shift operator and $\Delta = (1 - q^{-1})$ is the differencing operator. A key difference between GPC and GPCL is that $\{\xi(t)\}$ is assumed to be partially repeatable with GPCL, whereas with GPC it is assumed to be purely random. In the following derivation (just as with GPC), $T(q^{-1})$ is used in place of $C(q^{-1})$, and is considered a fixed design polynomial used for tuning GPCL (as described in Section 4.4). Hereinafter it will be assumed that $C(q^{-1}) = 1$. The CARIMA model is used to predict the future response of the plant. The free response predictions $f(t)$ are obtained in GPC (Clarke and Mohtadi, 1989) by convolving $T(q^{-1})/\Delta$ with $\{y^{*f}(t+j)\}$, where

$$y^{*f}(t+j) = \frac{\Delta y(t+j)}{T(q^{-1})} \quad \forall j \le 0, \qquad (2)$$

$$y^{*f}(t+j) = -\sum_{i=1}^{j} a_i y^{*f}(t+j-i) + \sum_{i=0}^{j-1} b_i u^{*f}(t+j-i) \quad \forall j \text{ with } 1 \le j < N_2, \qquad (3)$$

$$u^{*f}(t+j) = \frac{\Delta u(t+j)}{T(q^{-1})}, \qquad (4)$$

$$\Delta u(t+j) = 0 \quad \forall j \text{ with } 0 \le j < N_2 \qquad (5)$$

and N_2 is the maximum output horizon. In (3) the unknown