# Nonlinear Modeling and Control of Servo Pneumatic Actuators

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***Abstract*—Pneumatic actuators are low-cost, safe, clean, and exhibit a high power to weight ratio. In this paper a new modeling approach and control law for pneumatic servo actuators are presented. The nonlinear system model is developed using a combination of mechanistic and empirical methods. The use of novel bipolynomial functions to model the valve flow rates is shown to produce a more accurate solution than prior approaches. A novel multiple-input single-output nonlinear position control law is designed using the backstepping methodology. The stability analysis includes the effects of friction modeling error and valve modeling error. Experiments are conducted with 9.5-mm bore and 6.4-mm bore pneumatic cylinders, and four low-cost two-way proportional valves. In experiments with the 9.5-mm bore cylinder and a 1.5-kg moving mass, maximum tracking errors of $\pm 0.5$ mm for a 1-Hz sine wave trajectory, and steady-state errors within $\pm 0.05$ mm for an S-curve trajectory were achieved.**

***Index Terms*—Actuators, Lyapunov methods, nonlinear systems, position control, pneumatic systems.**

## I. Introduction

PNEUMATIC actuators continue to generate significant research interest due to their unique advantages. They are low-cost, safe, clean, and possess a high power to weight ratio. They have been successfully applied in rehabilitation [1], assistive devices [2], [3] and walking robots [4]. The lack of portability of their power source has also been addressed in recent research [5], [6].

In [5] solenoid injected monopropellant was used to power a 27-mm bore pneumatic cylinder. The cylinder was controlled using two 2-way solenoid valves and one 3-way custom servo valve. The system was shown to be portable, and more energy efficient than battery-powered servomotors. A free piston pneumatic compressor was proposed as a mobile robot power source in [6]. Their preliminary experimental results demonstrate the promise of this device.

Improvements in pneumatic control were reported in [7]–[9]. In [7] a rodless cylinder with a 25-mm bore was position controlled using a 4-way servo valve. Steady-state errors of $\pm 0.01$ mm and tracking errors within $\pm 1$ mm were achieved for a range of payloads and operating conditions using a sliding-mode control algorithm. Independent stiffness and force control was demonstrated in [8] using a 27-mm bore cylinder controlled by two three-way proportional valves and a multiple-input multiple-output sliding mode controller. A similar setup was used to effectively implement impedance control in [9].

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In the prior servo pneumatics research standard sized pneumatic cylinders were employed. Miniature sized pneumatic cylinders (with bore size less than 10 mm) have not been studied. These cylinders possess the advantages listed above and are applicable to smaller scale applications in robotics, such as robotic hands and millirobotics. Their small bore size produces three significant effects. First, it results in a faster pressure response that makes the closed-loop more stable. Second, the seal friction is roughly proportional to the bore circumference while the maximum pneumatic force is proportional to the piston area. So as the bore size is reduced the ratio of friction force to maximum pneumatic force increases. Third, the chamber pressures and piston position are more sensitive to small variations in the mass flow rate so the flow rate behavior of the valves must be modeled precisely. In our experience the latter two have the greatest effect and make precise tracking control more difficult with smaller bore sizes.

In this paper, the developments of a novel modeling approach and control law for pneumatic servo systems are presented. The system structure and model derivation is described in Section II. A multiple-input single-output (MISO) nonlinear controller is designed in Section III. While our modeling approach and control law can be applied to conventional sized pneumatic cylinders we have chosen to experimentally verify them with miniature cylinders. Experimental results for 9.5-mm bore and 6.4-mm bore cylinders are presented in Section IV. Conclusions are drawn in Section V.

## II. System Modeling

### A. System Structure

The system hardware is illustrated schematically in Fig. 1. Two double-acting cylinders were investigated. The first has a 9.5-mm diameter bore, 3.2-mm diameter rod, and 25.4-mm stroke (Clippard, model 3SD-T). The second was created by coupling together two single-acting cylinders (with springs removed) with 6.4-mm bores and 10-mm strokes (both are Clippard, model SM-6). For brevity, these cylinders will be hereafter referred to as the "9.5-mm cylinder" and the "6.4-mm cylinder," respectively, and are shown in Fig. 2. The system also includes four low-cost 2-way proportional valves (Clippard, model ET-P-05, US $50 each). These are labeled V1–V4 in the figure. The valves have an input range of 0–5 V. The air supply pressure $P_s$ is 0.65 MPa absolute. The four valves are interfaced through unity gain amplifiers to a PC via an analog/digital I/O board. The payload mass is mounted on a linear slide table. A linear variable displacement transformer (LVDT) position sensor is connected to the table to measure the displacement. The assembled 9.5-mm cylinder, LVDT, linear slide and payload are shown in Fig. 3. The position signal is sampled